DYNAMICAL EVOLUTION OF DUST PARTICLES IN THE KUIPER DISK

Elizabeth K. Holmes[(1)], Stanley F. Dermott[(2)], and Bo A. S. Gustafson[(2)]

[(1)]National Research Council Resident Research Associate, Jet Propulsion Laboratory, California Institute of Technology, M/S 169-506, 4800 Oak Grove Drive, Pasadena, CA 91109, USA, elizabeth.holmes@jpl.nasa.gov
[(2)]Department of Astronomy, University of Florida, Gainesville, FL 32611, USA, dermott@astro.ufl.edu, gustaf@astro.ufl.edu

ABSTRACT

A Kuiper belt dust disk will have a resonant structure, arising because the Plutinos are in the 3:2 mean motion resonance with Neptune. We run numerical integrations of particles originating from Plutinos to determine what percentage of particles remain in the resonance for a variety of particle and source body sizes. The dynamical evolution of the particles is followed from source to sink with Poynting-Robertson light drag, solar wind drag, radiation pressure, the Lorentz force, neutral interstellar gas drag, and the effects of planetary gravitational perturbations included. The number of particles in the 3:2 resonance increases with decreasing β for the cases where the initial source bodies are small and the percentage of particles in resonance is not significantly changed by either the addition of the Lorentz force, as long as the potential of the particles is small ($U \approx 5\ V$), or the effect of neutral interstellar gas drag.

1. INTRODUCTION

As of January 2002, more than 550 Edgeworth-Kuiper belt objects, or KBOs, have been discovered (18; 16), which translates into an estimated 1×10^5 KBOs with diameters greater than 100 km (11) orbiting with semi-major axes between 40 and 200 AU (18). In the asteroid belt, collisions between asteroids supply dust particles to the zodiacal cloud, the Sun's inner dust disk. By comparison, it has been postulated that collisions between KBOs could initiate a collisional cascade which would produce a Kuiper dust disk. A Kuiper dust disk will have a resonant structure, with two concentrations in brightness along the ecliptic longitude (3; 13) arising because an estimated 6% of the Kuiper belt objects are in the 3:2 mean motion resonance with Neptune (18). In order to study the dynamics of the Kuiper disk, we run numerical integrations of particles originating from source bodies trapped in the 3:2 resonance and we determine what percentage of particles are in the resonance for a variety of particle and source body sizes. Because the particle properties (shape, composition, etc.) are not well known, the variable we use to denote particle size is β, the magnitude of the ratio of the force of radiation pressure to the force of gravity. The dynamical evolution of the particles is followed from source to sink with Poynting-Robertson light drag (PR drag), solar wind drag, radiation pressure, the Lorentz force, neutral interstellar gas drag, and the effects of planetary gravitational perturbations included.

2. ANALYTICAL PREDICTIONS

Micron sized dust particles can be generated in the Kuiper belt by the breakup of Kuiper belt objects. The sizes of the source bodies and the sizes of the particles themselves determine whether the particles will become trapped in a particular resonance. For instance, particles generated from a parent body larger than $\sim$ 60 km in diameter will not all become trapped in the resonance, since the dispersion in initial semi-major axis of dust particles that escape from a 60 km diameter source body will be greater than the libration width, assuming the particles leave the source body with a velocity $\approx$ the escape velocity.

Trapping is also a function of β, or particle size. For particles less than $\sim$ 500 μm, radiation pressure plays a significant role. The location of the 3:2 mean motion resonance with Neptune, a', is given by Eq. 1 where a_n is the semi-major axis of Neptune.

$$\begin{aligned} a' &= a'_{\beta=0}\,(1-\beta)^{1/3} \\ &= a_n \left(\frac{3}{2}\right)^{2/3} (1-\beta)^{1/3} \end{aligned} \quad (1)$$

As β increases, a' decreases, as shown in Table 1. The particle diameters in the table are obtained by assuming the particles are spheres composed of astronomical silicate. The calculations in this paper are performed as a function of β, which allows for a wider assumption of particle sizes and shapes since β is highly dependent on particle mass, but less dependent on particle shape or structure (8). Particles with a high enough value of β will get "blown out" of the maximum libration width and will not be in resonance, at least initially. Due to PR drag, they could eventually spiral into orbits with smaller and

Table 1. Location of the 3:2 external mean motion resonance as a function of β.

Sphere Diameter (μm)	β (= $-F_{rad}/F_{grav}$)	a' (AU)
4	0.12928	37.61
7	0.07104	38.43
10	0.04868	38.74
13	0.03688	38.90
20	0.02343	39.08
50	0.00905	39.27
100	0.00446	39.33
...	0	39.39

smaller semi-major axes until they become trapped in the resonance, or they could become trapped in the other resonances that populate the inner Kuiper belt, such as the 2:1 or the 5:3 mean motion resonances. It is necessary to run numerical integrations to understand this complicated behavior; however we can make some analytical predictions.

Neglecting all forces except PR drag, radiation pressure, and gravity, we can calculate the minimum value of the eccentricity a particle must have to become trapped in the 3:2 resonance, as well as the smallest size particle (corresponding to a maximum value of β) that can be trapped (2; 19; 10). To find the minimum eccentricity a particle must have to become trapped in an external mean motion resonance, we equate the magnitude of the change in semi-major axis with time due to resonant trapping, $da'/dt|_{res}$, with the change in semi-major axis with time due to PR drag, $da'/dt|_{PR}$. Neglecting terms of order e'^2 and higher and solving for e' yields

$$e'_{min} \geq \frac{(GM_{\odot})^{1/2}}{f_d(\alpha)\,\mu_n\,c}\,\frac{2^{1/3}}{3^{4/3}}\,\frac{\beta}{a_n^{1/2}\,(1-\beta)^{2/3}} \qquad (2)$$

were $\mu_n = GM_n$, M_n is the mass of Neptune, G is the gravitational constant, c is the speed of light, and $M_{\odot}$ is the mass of the Sun. The variable $f_d(\alpha)$ is a function of the resonance, Laplace coefficients, and α, which is equal to a_n/a' for external mean motion resonances with Neptune (14). We can solve Eq. 2 for β in order to determine β_{max}. Since β is a function of particle size, β_{max} gives a limit on the smallest particles that can be trapped in resonance. Unfortunately, Eq. 2 can not be easily solved for β_{max} since it is not a simple function of β and since $f_d(\alpha)$ is a function of β. An approximate expression for β_{max} can be found. For $e' = 0.05, 0.13 \leq \beta_{max} \leq 0.64$, which corresponds to a minimum particle diameter, d_{min}, of roughly $1\mu m \leq d_{min} \leq 4\mu m$ assuming a spherical particle composed of astronomical silicate.

In the preceding discussion, we have neglected all forces except radiation pressure, gravity, and PR drag. However, other forces such as the Lorentz force and the effect of neutral interstellar gas drag are important to Kuiper dust grains. For instance, Lorentz forces become important for small particles at large heliocentric distances since the Lorentz force varies as $1/r$ while gravity and radiation pressure vary as $1/r^2$ (7). These additional forces are not incorporated in the theory, so instead they must be included in numerical simulations.

3. NUMERICAL INTEGRATIONS

We use the numerical integrator RADAU (4) to find the percentages of particles in resonance for a variety of initial conditions. RADAU is a very accurate integration code which employs the Runge-Kutta method. RADAU is very flexible, allowing the introduction of additional forces. The runs discussed in this paper have 249 particles, giving an accuracy in the estimate of the number of particles in resonance of $\sim \sqrt{N_r}/N_r$, where N_r is the number of particles in resonance. However, when N_r was zero, the error was estimated to be approximately zero. The integrations were run for 250,000 years and the orbital elements of the particles were output every 100 years so we could obtain an accurate variation of the resonant argument. The integrator considers PR drag, solar wind drag, radiation pressure, and the effects of the gravitational perturbations of seven planets. Mercury and Pluto are excluded from consideration since their masses are very low. In addition, in some of the runs, we have considered the effect of the Lorentz force on the particles as well as the effect of neutral interstellar gas drag. Runs have been performed for particles ejected from source bodies of 0 km, 10 km, and 100 km with a variety of sizes: 4, 10, 20, 50, 100 μm diameter spherical astronomical silicate particles (corresponding to β = 0.12928, 0.04868, 0.02343, 0.00905, and 0.00446) and the β = 0, or no drag case. We also run several cases for particles with β = 0.07104 and 0.03688, corresponding to 7 and 13 μm spherical particles.

3.1. Results for the Standard Forces Case

The percentage of particles in the 3:2 mean motion resonance with Neptune for the case including the effects of gravity, PR drag, solar wind corpuscular drag, and radiation pressure as a function of β is shown in Fig. 1. The open squares correspond to the case where the particles had no initial velocity dispersion, while the filled triangles and filled circles correspond to the cases where the particles were generated from 10 and 100 km diameter source bodies, respectively.

For the 0 km diameter source body case, the percentage of particles that remain in (or are later recaptured into) the 3:2 mean motion resonance roughly increases with decreasing β. The large error bars for the smaller particle sizes (higher β values) are due to the lower percentage of particles trapped for those cases, since we are assuming Poisson statistics and are quoting a very conservative estimate of the errors. A trend is evident, though, with no particles in resonance for the 4 μm case while nearly all (99.6 %) of the particles are in resonance for the β = 0 case. The runs with particles generated from 10 km

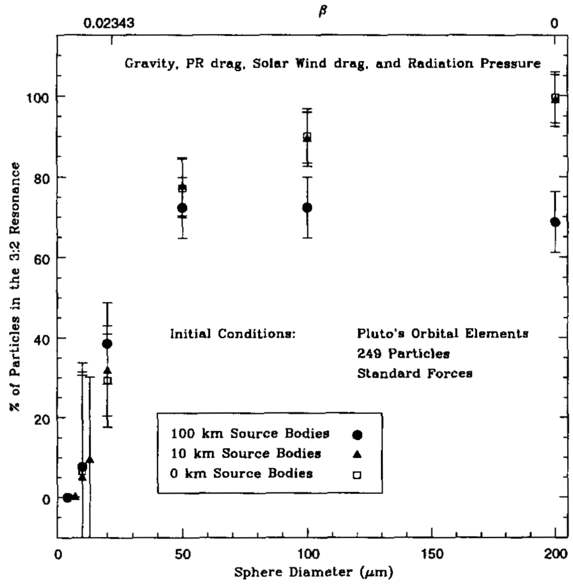

Figure 1. Percentage of particles in the 3:2 mean motion resonance with Neptune for the case including the effects of gravity, PR drag, solar wind corpuscular drag, and radiation pressure, as a function of β (particle size).

diameter source bodies also exhibit the trend that the percentage of particles that remain in (or are later recaptured into) the 3:2 mean motion resonance roughly increases with decreasing β, with no particles in resonance for either the 4 μm or 7 μm cases while nearly all (98.8 %) of the particles are in resonance for the $\beta = 0$ case. This is in agreement with the value of β_{max} of ≥ 0.13 ($d_{min} \leq$ 4 μm) calculated using Eq. 2.

The runs with particles generated from 100 km diameter source bodies were performed for cases with β values of 0.12928, 0.04868, 0.02343, 0.00905, 0.00446, and 0. For $\beta > 0.00905$, the percentage of particles in the 3:2 resonance roughly increases with increasing particle size. However, the percentage of particles in resonance levels off at a value ~ 71 % for $\beta \leq 0.00905$. This is in agreement with the analytical prediction given in Section 2, that particles generated from a parent body larger than ~ 60 km in diameter will not all become trapped in the resonance, since the dispersion in initial semi-major axis of dust particles that escape from a 60 km diameter source body will be greater than the libration width.

3.2. Lorentz Force Results

In the preceding runs, we have included gravity, radiation pressure, solar wind corpuscular drag, and PR drag. However, other forces, such as the Lorentz force and the effect of neutral interstellar gas drag, could potentially be important for dust grains in the Kuiper disk. Interplanetary dust particles are charged and a typical particle is expected to have a net positive potential, U, of $\approx 5\ V$ (5; 7). (However, this quantity is uncertain and could be as high as 100 V (12).) As a result of their charge, the particles are coupled to the solar wind driven interplanetary magnetic field. The force exerted on the particle by the interplanetary magnetic field, can be written as $\vec{F}_L = q\vec{v} \times \vec{B}$, which is the classical representation of the Lorentz Force (7). We have made a set of runs including the Lorentz force with $U = 5\ V$ that are otherwise identical to those presented in Fig. 1. Additionally, since U is uncertain, we have included some runs with $U = 20\ V$.

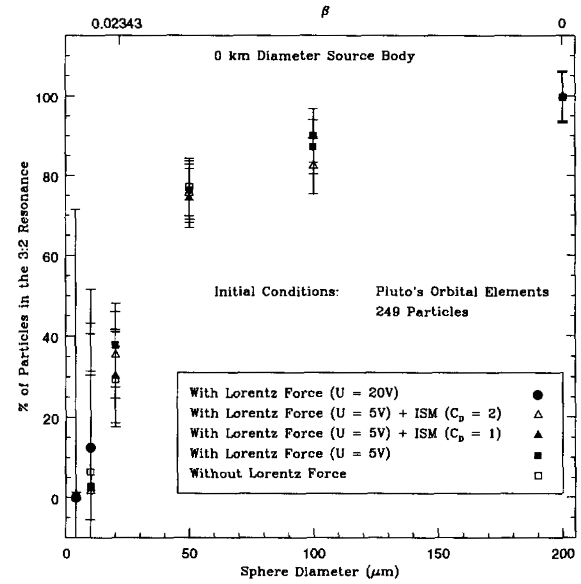

Figure 2. Percentage of particles in the 3:2 mean motion resonance with Neptune as a function of β, assuming particles were generated from a source body of 0 km in diameter in the 3:2 resonance. The open squares correspond to numerical runs with the effects of gravity, PR drag, solar wind corpuscular drag, and radiation pressure included, that is, the "standard forces". The filled squares and filled circles correspond to runs which include, in addition to the standard forces, the Lorentz force with a potential U equal to 5 and 20 Volts, respectively. The filled and open triangles correspond to runs which include the standard forces, the Lorentz force with $U = 5\ V$, and the effects of neutral interstellar gas drag assuming $C_D = 1$ and 2, respectively.

The results of the integrations are shown in Fig. 2 for particles generated from 0 km source body, that is, with no initial velocity dispersion. Plotted are the percentages of particles in the 3:2 mean motion resonance with Neptune versus β. The addition of the Lorentz force with $U = 5\ V$ does not have a significant effect on whether the particles remain in the 3:2 resonance. The results of the runs that include the Lorentz force with a U of 5 V for sources with 10 km and 100 km diameter source bodies (not shown) are similar, in that the Lorentz force does not significantly change the percentage of particles in resonance from those shown in Fig. 1. However, for small particles, $\sim 10\ \mu m$ in diameter, the Lorentz force can have an appreciable effect, especially for a high potential. We performed numerical simulations with U = 20 V. This potential is probably too high for grains in the Kuiper belt, but it does serve as an interesting test case for the effects of the Lorentz force (9).

3.3. Neutral Interstellar Gas Drag Results

This last force, while small in magnitude, acts from a specific direction and could potentially cause noticeable effects on the orbits of dust particles over a large period of time. The Ulysses spacecraft has detected a stream of interstellar helium, a tracer of hydrogen, originating from an ecliptic longitude of 252° and an ecliptic latitude of +2.5° moving with a speed of 26 km/s (20; 6), which is in good agreement with the direction of motion of the local cloud in which the Sun is embedded with respect to the solar system (1). These neutral gas atoms can collide with dust grains in the solar system, causing a change in the momentum of the dust particles. While the effects of one collision are negligible, over a period of time repeated collisions have the potential to alter the orbital elements of the dust particle significantly (17).

In Fig. 2, a full set of runs (for 4, 10, 20, 50, and 100 μm diameter particles as well as the $\beta = 0$ case) with the effects of gravity, PR drag, solar wind corpuscular drag, radiation pressure, the Lorentz force ($U = 5$ V) and the effects of neutral interstellar gas drag included. For the case of neutral interstellar gas drag, performed two sets of runs for different values of C_D, the free molecular drag coefficient due to hydrogen atoms impacting a spherical dust particle (7; 17). The results are not statistically different than the basic case which does not include the Lorentz force or neutral gas drag (9), in contrast to the predictions of (17).

4. DISCUSSION

From Fig. 2, it is clear that the probability that the particles remain in (or are later recaptured into) the 3:2 mean motion resonance increases roughly with decreasing β (i.e., increasing particle size). Consequently, a size-frequency distribution for a Plutino disk must be weighted toward the larger size particles. In addition, as long as the potential, U, of the particles is small ($U \approx 5$ V), the addition of either the Lorentz force or neutral interstellar gas drag does not change the percentage of particles in resonance significantly.

ACKNOWLEDGMENTS

This research was funded in part by NASA GSRP grant. Part of the research described in this publication was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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